

# **The Physics of energy**

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Introduction to the notion of **energy**.  
The energy at the microscopic scale and  
the **entropy**

# Energy

## Energy

In physics, energy (Ancient Greek: ἐνέργεια *energeia* "activity, operation") is an indirectly observed quantity that is often understood as the ability of a physical system to do work on other physical systems

Capability of doing WORK...  $WORK = FORCE \times Displacement$

*"It is important to realize that in physics today, we have no knowledge what energy is."*



Richard Feynman, in *The Feynman Lectures on Physics* (1964) Volume I, 4-1

# Energy: a hint of history

The *vis viva* (living force), which **Gottfried Leibniz** defined as the product of the mass of an object and its velocity squared; he believed that total *vis viva* was conserved.



To account for slowing due to friction, Leibniz theorized that thermal energy consisted of **the random motion of the constituent parts of matter**, a view shared by **Isaac Newton**, although it would be more than a century until this was generally accepted.



# Energy: a hint of history

The **conservation of energy** was proposed by [Gottfried Leibniz](#) over the period 1676–1689, the theory was controversial as it seemed to oppose the theory of [conservation of momentum](#) advocated by Sir [Isaac Newton](#) and [René Descartes](#). The two theories are now understood to be complementary.



In 1807, **Thomas Young** was possibly the first to use the term "energy" instead of vis viva, in its modern sense.

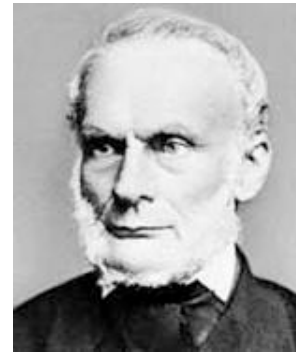
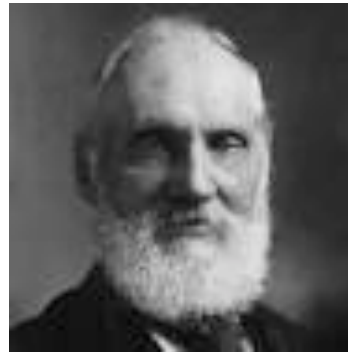


# Energy: a hint of history

**Gustave-Gaspard Coriolis** described "kinetic energy" in 1829 in its modern sense.

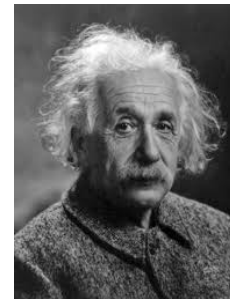
In 1853, **William Rankine** coined the term "potential energy".

William Thomson (Lord **Kelvin**) amalgamated all of these laws into the laws of thermodynamics, which aided in the rapid development of explanations of chemical processes by **Rudolf Clausius, Josiah Willard Gibbs, and Walther Nernst.**



# Energy: a hint of history

It also led to a mathematical formulation of the concept of entropy by Clausius and to the introduction of laws of radiant energy by **Jožef Stefan**.



**Albert Einstein** proposed mass–energy equivalence in 1905 in a paper entitled "Does the inertia of a body depend upon its energy-content?".

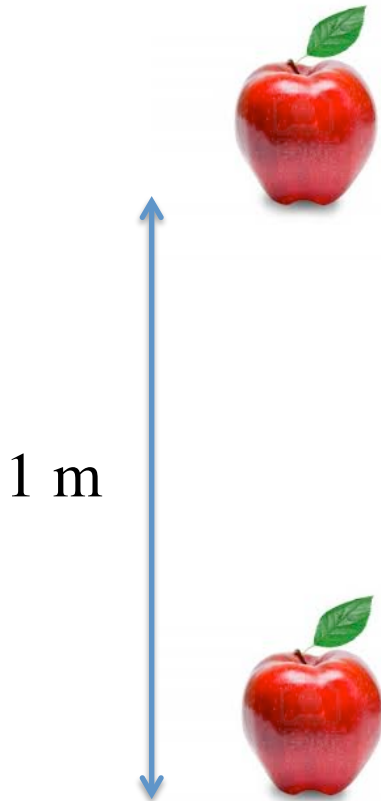
Since 1918 it has been known that the law of conservation of energy is the direct mathematical consequence of the translational symmetry of the quantity conjugate to energy, namely time (**Emmy Noether**).



# Use of energy

Unit of measure: Joule = 1 J = 1 N x 1 m

Power = Energy /time

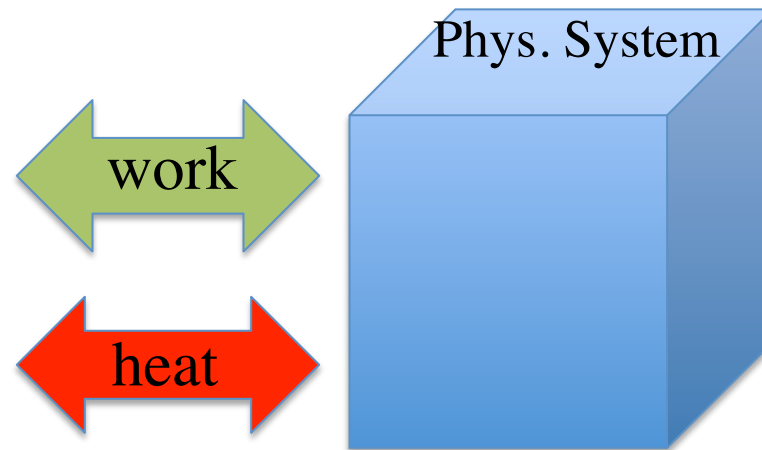


Task	Power (W)
Average power of a Boeing 747 airplane	$10^8$
Full power aircraft fighter	$10^6$
Full power car engine	$10^5$
Operate a microwave oven	$10^3$
Being alive for an average adult human	$10^2$
Brain functioning for an average human	10
<u>mobile</u> phone calling	1
Emission of a standard WI-FI router	$10^{-1}$
Functioning of a LED light	$10^{-2}$
Functioning of a miniature FM receiver	$10^{-3}$
Functioning o a wireless sensor node	$10^{-4}$
Low power radio module	$10^{-5}$
Functioning of a quartz wristwatch	$10^{-6}$
Operation of a quartz oscillator	$10^{-7}$
Sleep mode of a microcontroller	$10^{-8}$
1 bit information erasure at room T (min)	$10^{-21}$



# Energy

The energy content of a system can be changed via exchanges of work and heat but there are some limitations...

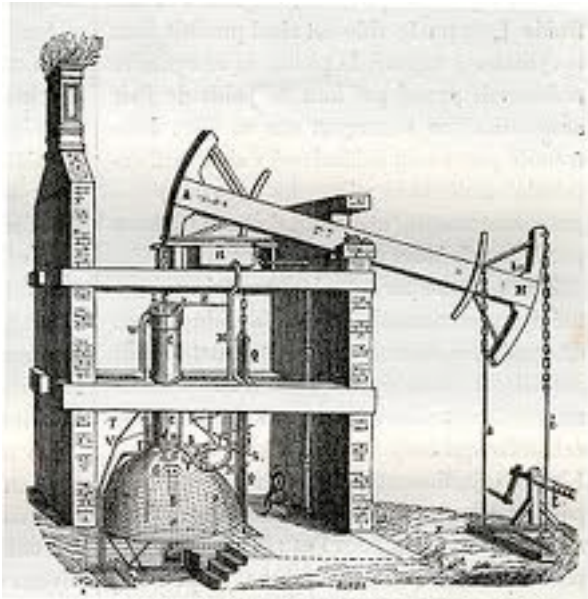


# Energy

Energy is a property of physical systems that can be used to perform work and usually comes inside physical objects like a **hot gas** or a **gasoline tank**.

Thinking about it we can ask questions like:

- how can we make the energy contained in a litre of gasoline to push forward a car
- how can we use the heat produced by burning coal to make the train run?

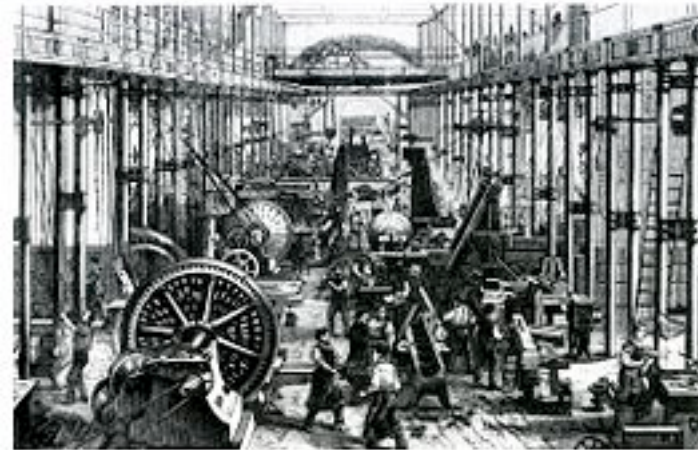
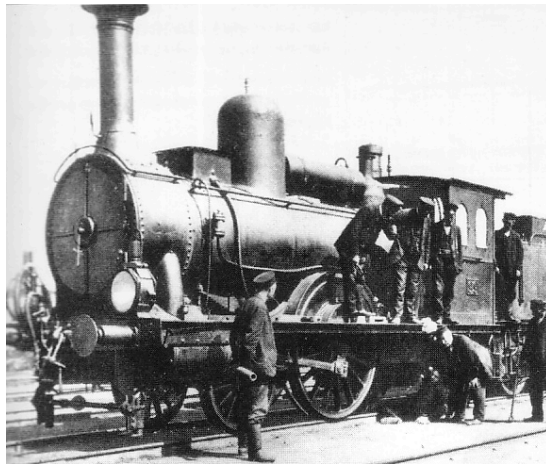


Questions like these were at the very base of the activities performed in the early seventeenth century by the first inventors of the so-called thermal machines.

People like **Thomas Newcomen** (1664-1729) who built the first practical steam engine for pumping water and **James Watt** (1736-1819) who few decades after proposed an improved version of the same machine.

# Thermal Machines

It is thanks to the work of scientists like **Sadi Carnot** (1796-1832) and subsequently of **Émile Clapeyron** (1799 - 1864), **Rudolf Clausius** (1822 - 1888) and **William Thomson** (Lord Kelvin) (1824 – 1907) that studies on the efficiency of these machines aimed at transforming heat (just a form of energy) into work brought us the notion of entropy and the laws of thermodynamics.



These laws do not tell us much about what energy is but they are very good in ruling what can we do and what we cannot do with energy. Let's briefly review them.

**The first law** of thermodynamics states that **the total energy of an isolated physical system is conserved during any transformation the system can go through.**

It was initially formulated in this form by Julius Robert von Mayer (1814 - 1878) and subsequently reviewed by James Prescott Joule (1818-1889) and Hermann Ludwig Ferdinand von Helmholtz (1821-1894).

**The second law** states that there are limitations to how much work we can get from a given amount of energy present in the form of heat. There exist different formulations that are all equivalent.

The two most popular are ascribed to Clausius and Kelvin:

Clausius formulation: “**No process is possible whose sole result is the transfer of heat from a body of lower temperature to a body of higher temperature**”.

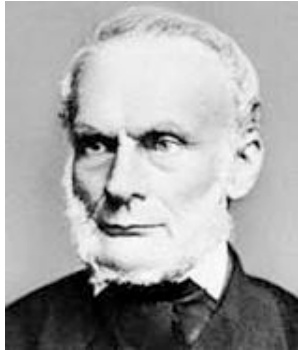
Kelvin formulation: “**No process is possible in which the sole result is the absorption of heat from a reservoir and its complete conversion into work**”.

An important consequence of the second law is that there is a limit to **the efficiency of a thermal machine**. This limit was discovered by Sadi Carnot in 1824 when he was only 28. He introduced the concept of thermal machine, generalizing the concept popular at that time of “steam engine”, and showing that the efficiency of any thermal machine operating between two temperatures is bounded by a quantity that is a function of the two temperatures only.



Few years after the work of Carnot, Clausius used this result to introduce a quantity that is useful in describing how much heat can be changed into work during a transformation. He proposed the name “**entropy**” for his quantity.

# ENTROPY



Clausius proved a theorem that states that during a **cyclic transformation**, if you do the transformation carefully enough not to lose any energy in other ways (like friction), then **the sum of the heat exchanged with the external divided by the temperature at which the exchange occurs is zero:**

$$\oint \frac{dQ}{T} = 0$$

This is equivalent to say that it exists a state function  $S$  defined as

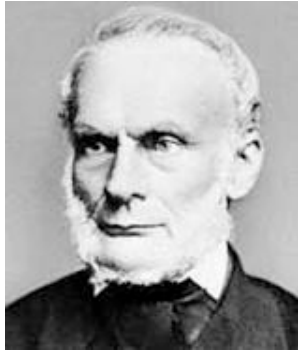
$$S_B - S_A = \int_A^B \frac{dQ}{T}$$

If you are not careful enough and you lose energy during the transformation then the inequality holds instead:

$$\oint \frac{dQ}{T} \leq 0$$

A transformation like this is also called an *irreversible transformation*

$$S_B - S_A \geq \int_{A \text{ irr}}^B \frac{dQ}{T}$$



$$S_B - S_A = \int_A^B \frac{dQ}{T}$$

The quantity of heat  $Q$  that appears in the Clausius equation is the amount of energy that goes into the increase of entropy.

It is useful to interpret the quantity  **$TdS$  as the amount of heat** (meaning thermal energy) **that cannot be used to produce work**. In other words during a transformation, even if we are carefully enough not to waste energy in other ways, we cannot use all the energy we have to do useful work. Part of this energy will go into producing an increase of the system entropy. *If we are not carefully enough the situation is even worse and we get even less work.*



This is sometimes accounted by the introduction of the so-called **Free energy**. The concept of Free energy was proposed by Helmholtz in the form:  $F = U - TS$ . The free energy  $F$  measures the maximum amount of energy that we can use when we have available the internal energy  $U$  of a system.



# Summary

## Energy

Capability of doing WORK...  $\text{WORK} = \text{FORCE} \times \text{Displacement}$   
Energy can be **changed** through flux of **work** and **heat**.

**Energy is conserved** (First Principle)

## Entropy

Measures the capability of change...

**Entropy increases in spontaneous transf.** (Second Principle)

**Equilibrium** is the competition of the tendency of Energy to reach a minimum and Entropy to reach a maximum (minimum Gibbs free energy).

$$F = U - T S$$

## Energy and Entropy: the microscopic interpretation

In general the entropy remained an obscure quantity whose physical sense was (and somehow still is) difficult to grasp.

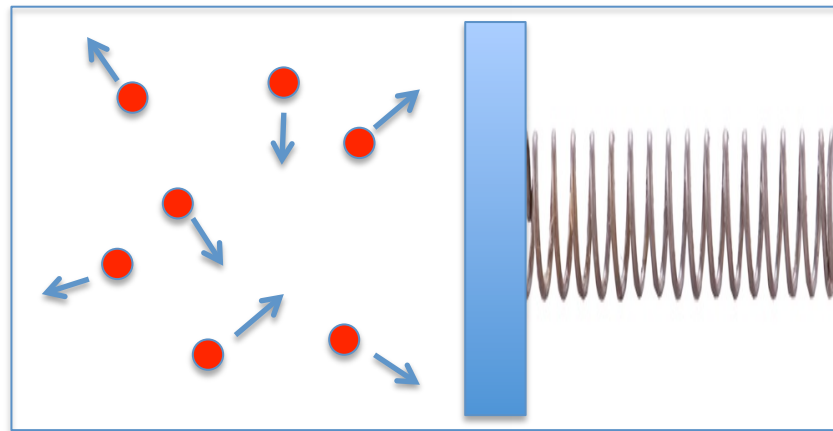
It was the work of Ludwig Boltzmann (1844 – 1906) that shed some light on the microscopic interpretation of the second law (and thus the entropy).

If a system can be in a number of different states, all compatible with the conservation laws, then it will evolve in order to attain the equilibrium condition identified with the **most probable state** among all the possible states it can be in.



To grasp the meaning of entropy at small scales...

The ideal world of Boltzmann is made by physical systems constituted by many small parts represented by colliding small spheres



Each sphere has the same mass  $m$  and velocity  $v$

Consider the two cases...

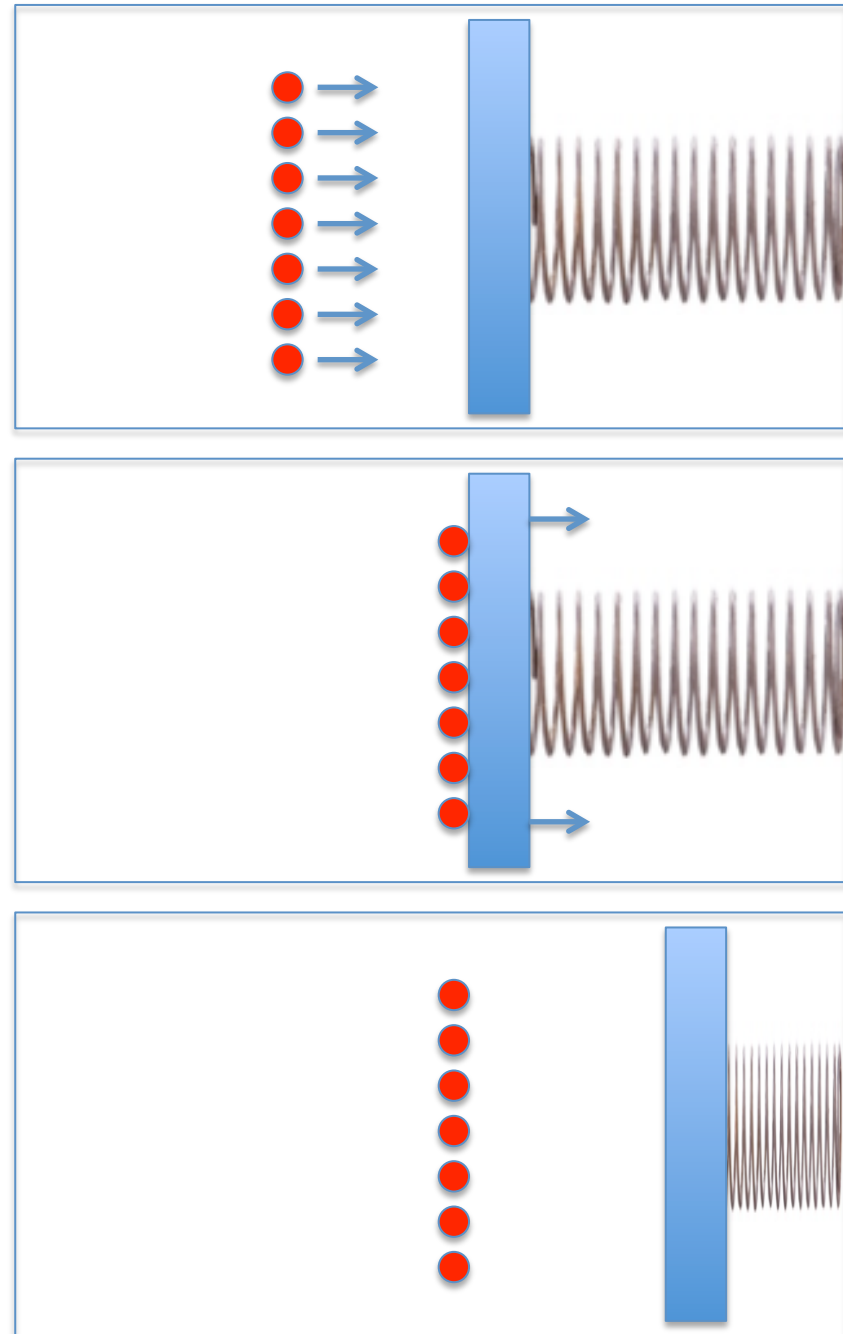
## First case

Let's suppose that these particles are contained in a box that has a moving set of mass  $M = Nm$ . The set is connected to a spring of elastic constant  $k$ , as in the figure, and is at rest.

If all the particles have the same velocity  $v$  and collide perpendicularly with the moving set at the same time, they will exchange velocity with the set. This will compress the spring up to an extent  $x_1$  such that

$$\frac{1}{2} M v^2 = \frac{1}{2} k x_1^2 = U$$

We can always recover the potential energy  $U$  when we desire and use it to perform work. In this case we can completely transform the energy of the gas particle into work.

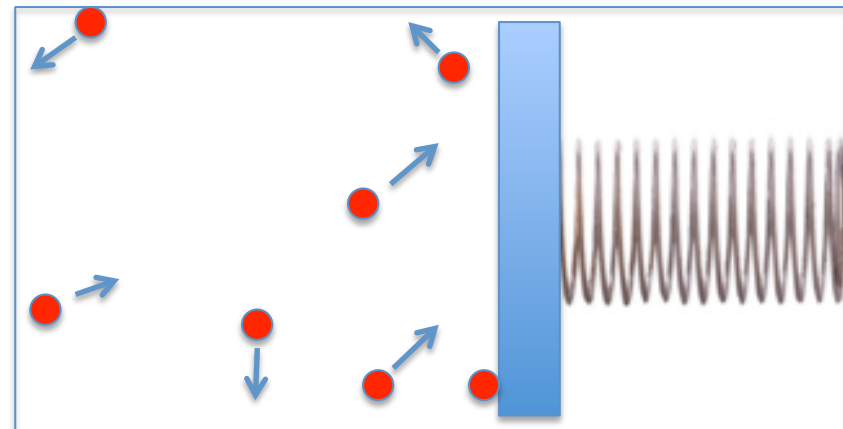
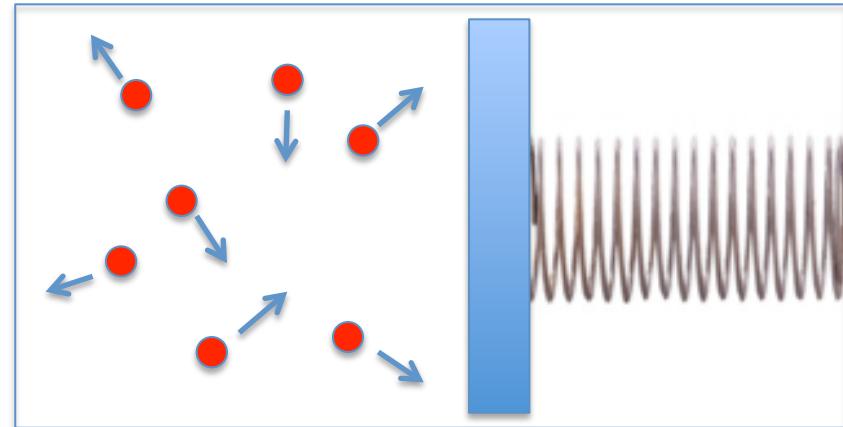


## Second case

What is on the contrary the most probable configuration for the particle in the gas? Based on our experience (and on some common sense as well) it is the configuration in which all the particles, although each with the same velocity  $v$ , are moving with random direction in the box.

The **energy of the gas is still the same (so is its temperature  $T$ )** but in this case the set will be subjected at random motion with an average compression of the spring such that its average energy is  $U/N$ .

**This is also the maximum work that we can recover from the potential energy of the movable set.**

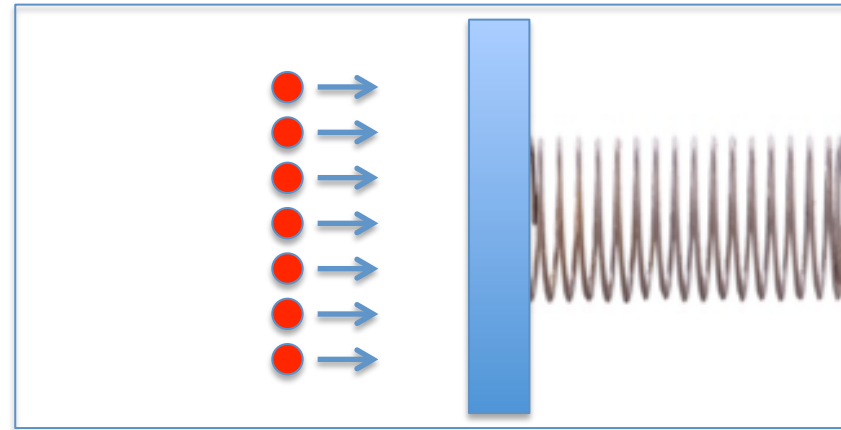


According to the definition of Free energy, the quantity that limits our capability of performing work is the entropy.

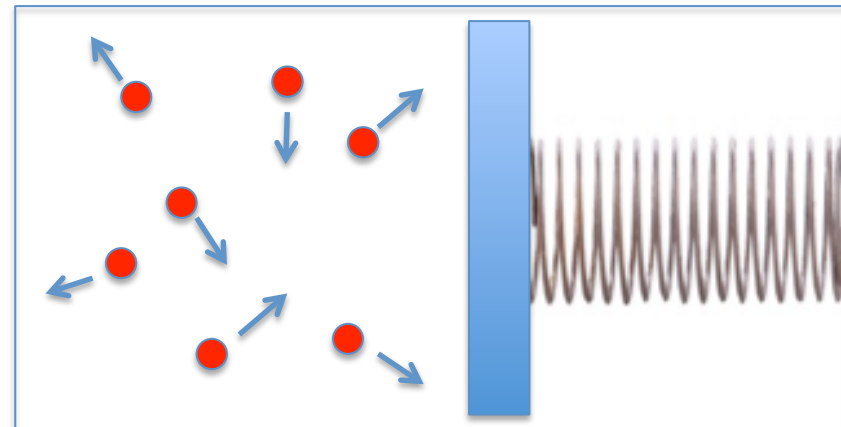
**Thus the systems that have the smaller entropy have the larger capability of performing work.**

*Accordingly we can use the entropy to put a label on the energetic content of a system.*

Two systems may have the same energy but the system that has the lower entropy will have the “most useful” energy.



low entropy



high entropy

## The microscopic perspective

This example helped us to understand how energy and entropy are connected to the microscopic properties of the physical systems.

In the simple case of an ideal gas, the system energy is nothing else than the sum of all the kinetic energies of the single particles.

**We can say that the energy is associated with “how much” the particles move.**

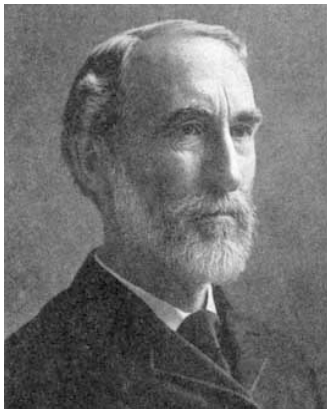
On the other hand we have seen that there is also a “quality” of the motion of the particles that is relevant for the entropy.

**We can say that the entropy is associated with “the way” the particles moves.**

This concept of “way of moving” was made clear by Boltzmann at the end of 1800, who proposed for the entropy the following definition:

$$S = K_B \log W$$

where  $K_B$  is the famous Boltzmann constant and  $W$  is also called the “number of configurations” and represents the number of ways we can arrange all the particles in the system without changing its macroscopic properties.



# Entropy

We define *shape entropy* the quantity

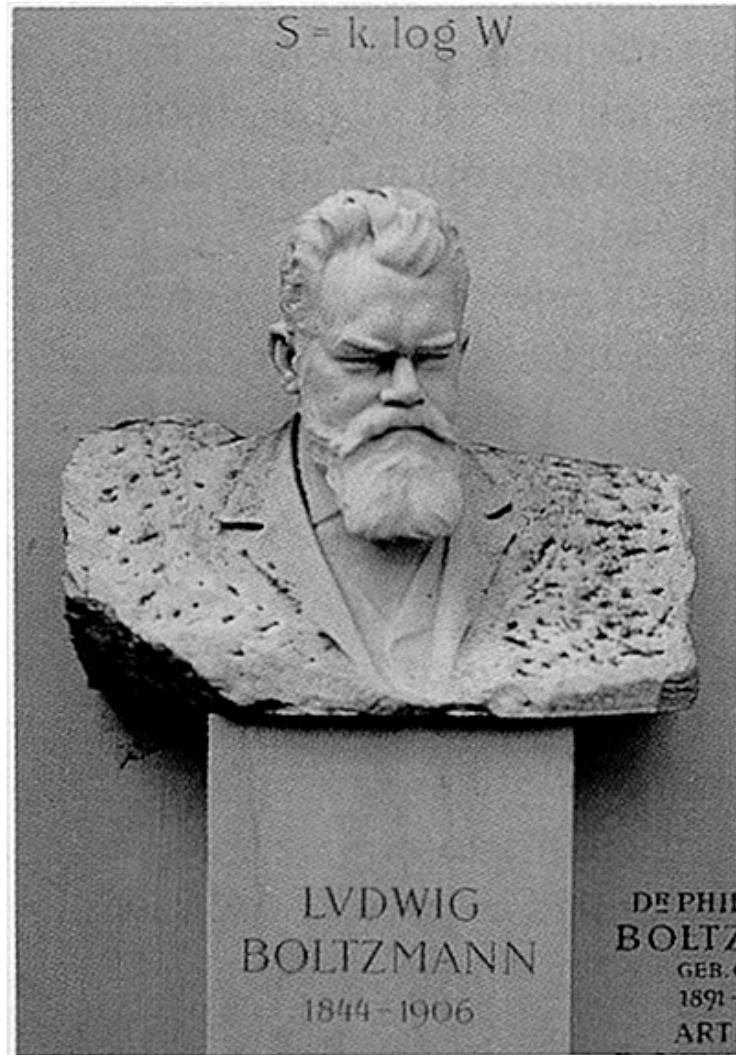
$$S_i = K \ln N_i$$

where  $K$  is an arbitrary constant. This quantity coincides with the microscopic form given by Boltzmann and Gibbs of the thermodynamic entropy initially introduced by Clausius, if we interpret the number of configurations  $N_i$  for a given shape as the number of accessible microstates for a given state of the thermodynamical system. Specifically, Gibbs entropy is given by

$$S_G = -K \sum_l p_l \ln p_l$$

$p_l$  is the probability of the microstate of index  $l$  and the sum is taken over all the microstates.





If the probability of the microstates are all the same, then the Gibbs entropy reduces to the Boltzmann entropy.

Thus if we identify the microstate of a physical object with a configuration that realizes one shape we have that the shape entropy IS the Boltzmann entropy of our object.

## What does irreversible mean?

During an irreversible transformation the entropy always increases.

Moreover, due to the Clausius *inequality* it always increases some more compared to what it would be required by the second law.

$$S_B - S_A \geq \int_{A \text{ irr}}^B \frac{dQ}{T}$$

Why is that? The answer is that in addition to the *physiological* increase there is an extra contribution due to the *dissipative effect* of the non-equilibrium processes. With *dissipative effect* we intend a way in which some low-entropy energy is changed into high-entropy energy. A typical example of dissipative process is friction.

**FRICION**

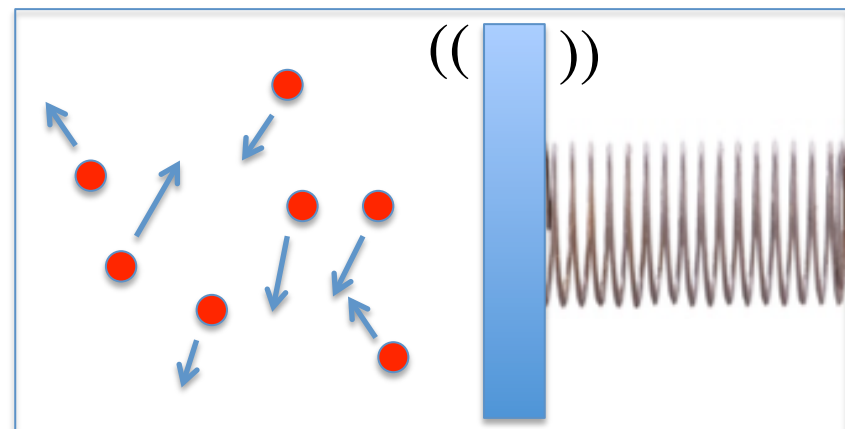
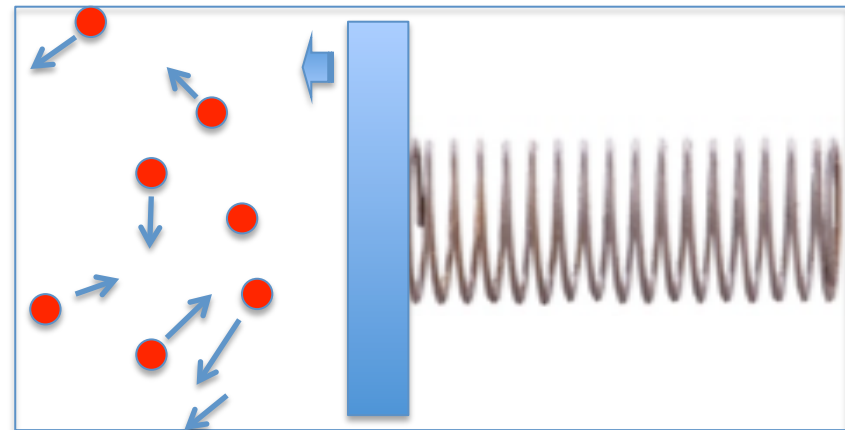
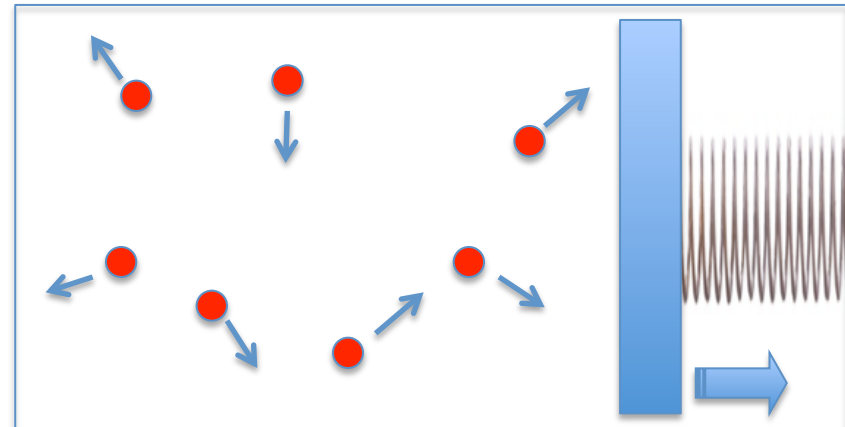
Consider the two cases...

## First case

We compress the spring to some extent and then we release the compression leaving it free to oscillate.

After few oscillations we observe that the oscillation amplitude decreases as a consequence of what we call the friction (viscous damping force) action due to the presence of the gas. The decrease ceases when the oscillation amplitude reaches a certain equilibrium value and after that it remains constant (on average).

Some energy has been dissipated into heat.



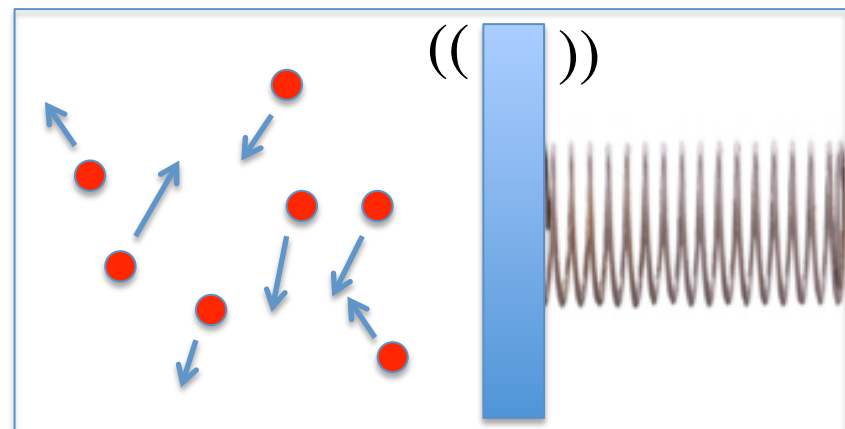
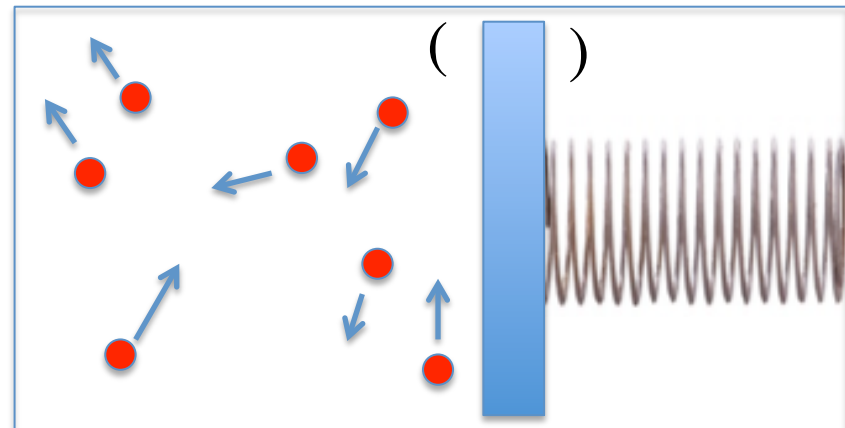
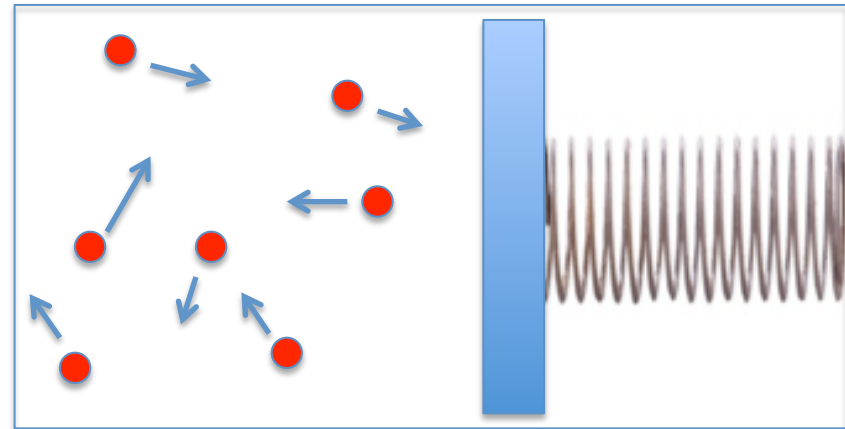
## Second case

We now start with the movable set at rest and leave it free.

After few seconds we will see that the set starts to move with increasing oscillation amplitude that soon reaches **an equilibrium condition** at the very same value (on average) of the first case.

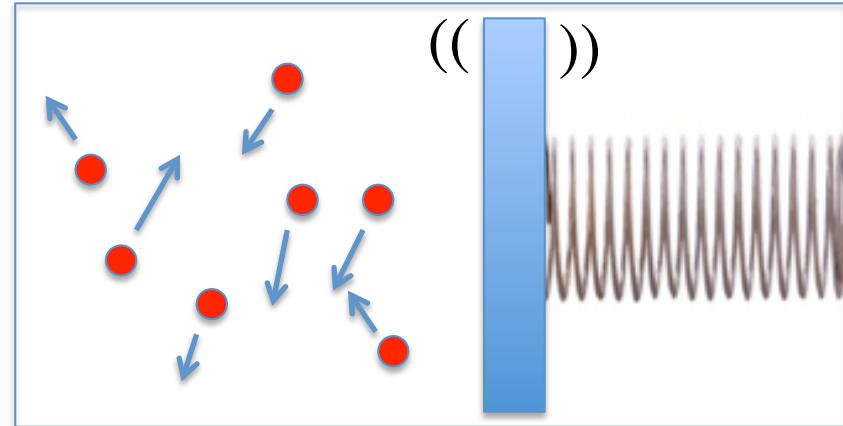
In both cases the two different roles of damping-force and pushing-force has been played by the gas.

This fact led to think that there must be a **connection between the process of dissipating energy** (a typical irreversible, i.e. non-equilibrium process) and the process of **fluctuating at equilibrium** with the gas.



# Fluctuation-Dissipation connection

This fact led to think that there must be a **connection between the process of dissipating energy** (a typical irreversible, i.e. non-equilibrium process) and the process of **fluctuating at equilibrium** with the gas.



In order to unveil such a link we need to introduce a more formal description of the dynamics of the movable set.

This problem has been addressed and solved by Albert Einstein (1879 - 1955) in his 1905 discussion of the Brownian motion and subsequently by **Paul Langevin** (1872 - 1946) who proposed the following equation:

$$m\ddot{x} = -m\gamma\dot{x} - \frac{dU}{dx} + \xi(t)$$

## Fluctuation-Dissipation connection

$$m\ddot{x} = -m\gamma\dot{x} - \frac{dU}{dx} + \xi(t)$$

$\xi(t)$  is the random force that accounts for the incessant impact of the gas particles on the set, assumed with zero mean, Gaussian distributed and with a flat spectrum or, delta-correlated in time (white noise assumption):

$$\langle \xi(t_1)\xi(t_2) \rangle = 2\pi G_R \delta(t_1 - t_2)$$

$G_R$  accounts for the fluctuation intensity. There must be a connection with the dissipation  $\gamma$ .

This relation has been established within the linear response theory (that satisfies the equipartition of the energy among all the degrees of freedom) initially by **Harry Theodor Nyquist** (1889 - 1976) in 1928, and demonstrated by **Callen and Welton** in 1951.

$$G_R = \frac{mK_B T}{\pi} \gamma$$

# Fluctuation-Dissipation Theorem **FDT**

Why is FDT important?

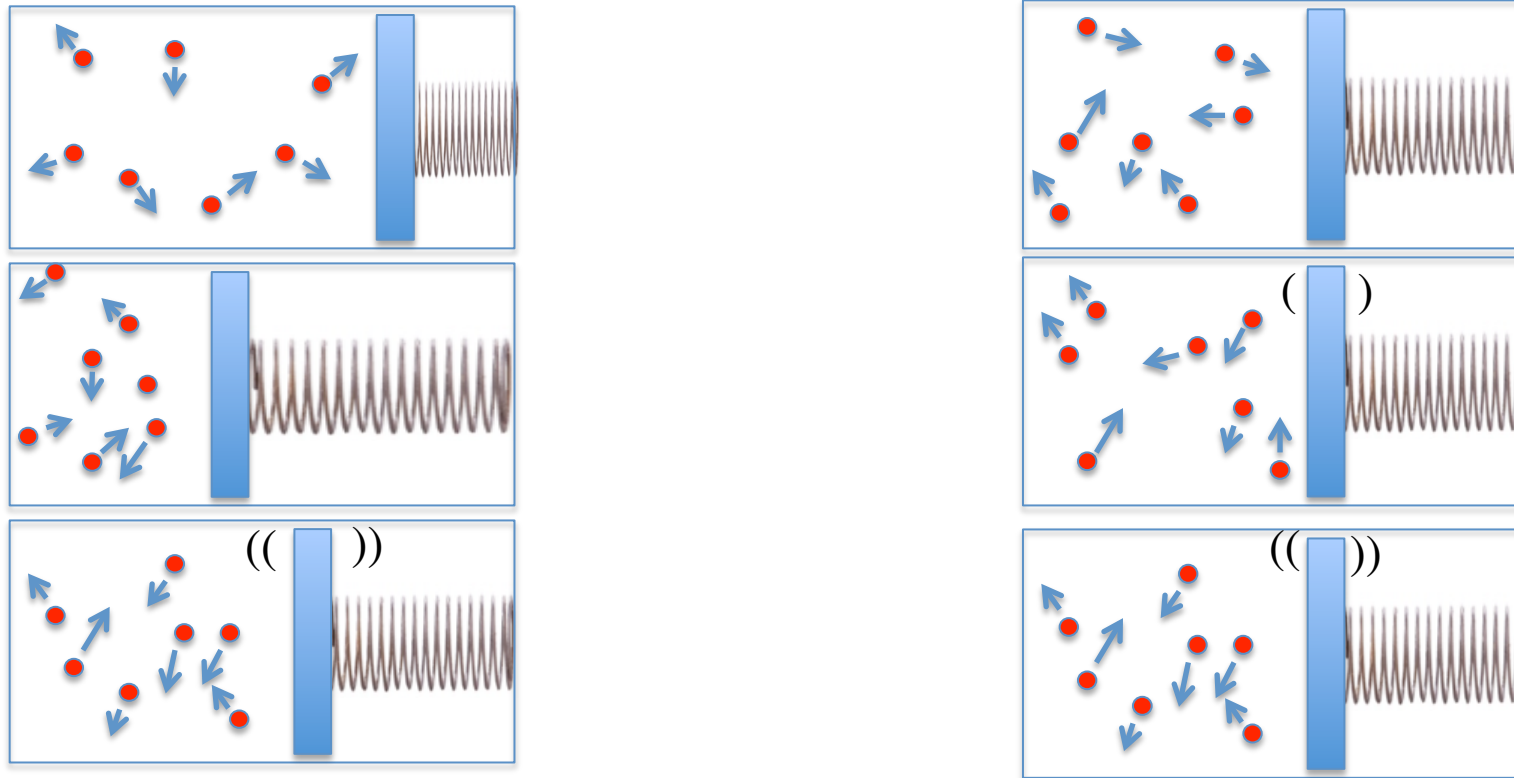
It is important because it represent an ideal bridge that connects:

**the equilibrium properties of our thermodynamic system**  
(represented by the amplitude and character of the fluctuations)

with

**the non-equilibrium properties**  
(represented here by the dissipative phenomena due to the presence of the friction).

# ENTROPY production – FLUCTUATION CONNECTION



dissipative properties = the capacity to produce entropy, are intrinsically connected to the equilibrium fluctuations.



To learn more:

**Energy Management at the Nanoscale**

L. Gammaitoni

in the book "ICT - Energy - Concepts Towards Zero - Power Information and Communication Technology" InTech, February 2, 2014